UNIVERSITY of HOUSTON

Evidence accumulation and change rate inference in dynamic environments

Joint lab meeting

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08/17/17

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Modeling perceptual decisions

world.png

Random dots reversal task

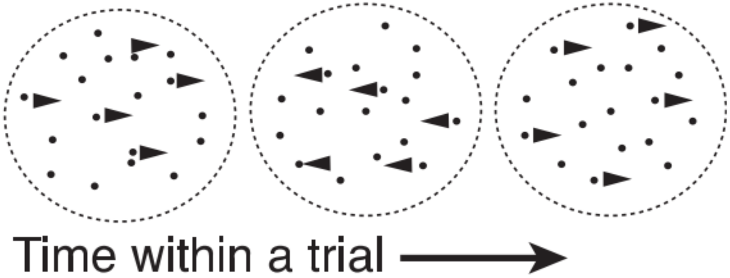


Figure from **?**

Hidden Markov Model

HMM1.png

Click task

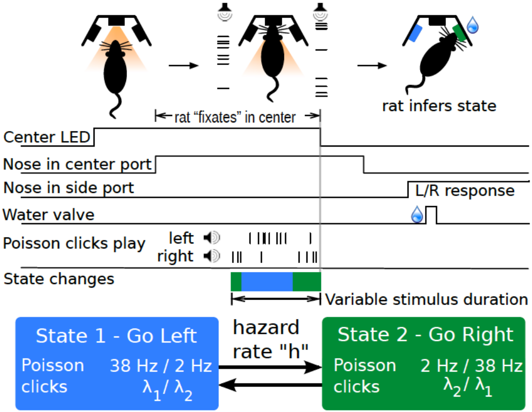
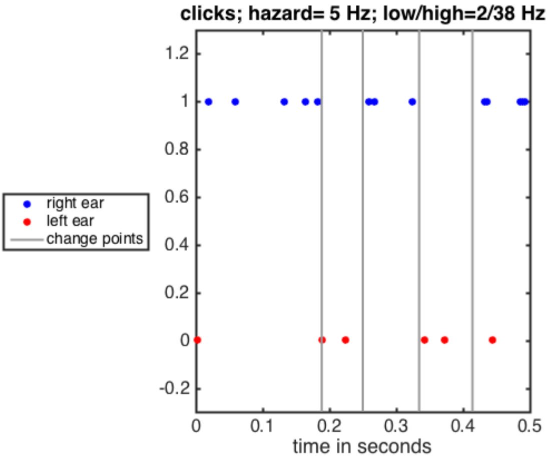


Figure from **?**

Example trials



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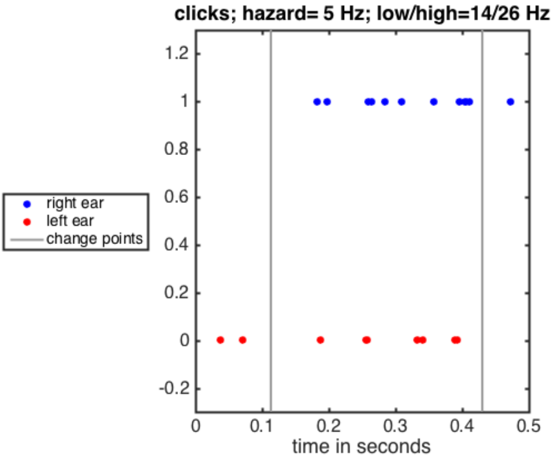

Evidence accumulation

- *decision variable*, $y_t = \ln P(H^+|\xi_{1:t})/P(H^-|\xi_{1:t})$, represents accrued *evidence* in favor of either choice.

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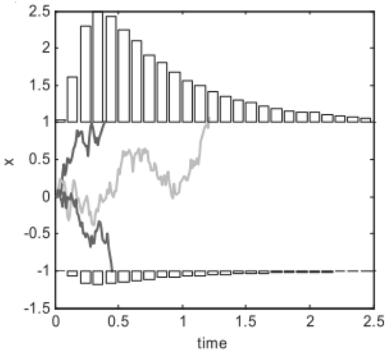
- *decision rule*:
  - *free response*: decide when threshold is reached, $|y_t| \geq \theta$
  - *interrogation*: decide at time T based on the sign of y_t

Evidence accumulation

Static environment: $H_t \equiv$ const. $\Rightarrow$ weight all observations equally



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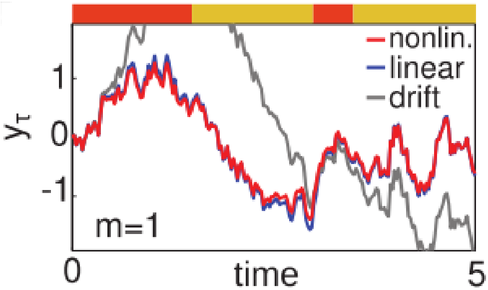
Evidence accumulation

Dynamic environment: $\{H_t\}$ is a Markov Chain $\Rightarrow$ discount old evidence



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How to deal with unknown hazard rate?

$\rightarrow$ track *change point count* a_t

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Recursive update equation – ?

$$P_n\left(H^{\pm},a\right) \propto P\left(\xi_n|H^{\pm}\right)\left[\left(1-\hat{h}_{n-1}(a)\right)\cdot P_{n-1}\left(H^{\pm},a\right) + \hat{h}_{n-1}(a-1)\cdot P_{n-1}\left(H^{\mp},a-1\right)\right]$$

Continuous-time approximation

Set: $\bar{P}_t^{\pm} := \sum_a P_n(H^{\pm}, a)$ and $\bar{A}_t^{\pm} := \frac{1}{t+\beta} \sum_a (a+\alpha) P_n(H^{\pm}, a)$

Moment closure – ?

$$d\bar{P}_t^{\pm} = \bar{P}_t^{\pm} \left[\left(g^{\pm}(t) + \frac{1}{2} \right) dt + dW^{\pm} \right] + \left[\bar{A}_t^{\mp} - \bar{A}_t^{\pm} \right] dt$$

$$d\bar{A}_t^{\pm} = \bar{A}_t^{\pm} \left[\left(g^{\pm}(t) + \frac{1}{2} \right) dt + dW^{\pm} \right] + \left(\bar{A}_t^{\mp} - \bar{A}_t^{\pm} \right) \left(\frac{1}{t+\beta} + \bar{A}_t^{\mp} + \bar{A}_t^{\pm} \right) dt$$

Click task

Evidence accumulation ODE - known h

$$\frac{dy_t}{dt} = \kappa \sum_{i\in I, j\in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2h \sinh(y_t)$$

Click task

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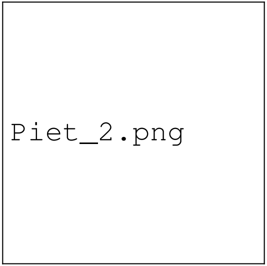

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But with unknown hazard rate?

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Likelihoods summary

$$f^+_{\Delta t}(01) = f^-_{\Delta t}(10) = \lambda_{\text{high}}\Delta t + o(\Delta t)$$
$$f^+_{\Delta t}(10) = f^-_{\Delta t}(01) = \lambda_{\text{low}}\Delta t + o(\Delta t)$$
$$f^+_{\Delta t}(11) = f^-_{\Delta t}(11) = o(\Delta t)$$
$$f^+_{\Delta t}(00) = f^-_{\Delta t}(00) = 1 - (\lambda_{\text{low}} + \lambda_{\text{high}})\Delta t + o(\Delta t)$$

Revisit discrete-time update equation

Set $x_{t_n}^{\pm}(a) := \log P_n(H^{\pm}, a)$, and,

$$\hat{h}_n(a) := \frac{\alpha + a}{\beta + \Delta t \cdot n} = \frac{\alpha + a}{\beta + t_n}$$

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Set $x_{t_n}^{\pm}(a) := \log P_n(H^{\pm}, a)$, and,

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Update equation

$$\Delta x_{t_n}^{\pm}(a) = \log \frac{P(\xi_{1:n-1})}{P(\xi_{1:n})} + \log f_{\Delta t}^{\pm}(\xi_n) \cdots$$
$$+ \Delta t \cdot \hat{h}_{n-1}(a-1) e^{x_{t_{n-1}}^{\mp}(a-1) - x_{t_{n-1}}^{\pm}(a)} - \Delta t \cdot \hat{h}_{n-1}(a)$$

Problematic continuum limit

When ξ_n corresponds to a single click $(10, 01)$, the limit,

$$\lim_{\Delta t \to 0} \left(\log \frac{P(\xi_{1:n-1})}{P(\xi_{1:n})} + \log f^{\pm}_{\Delta t}(\xi_n) \right),$$

is hard to take, since $f^{\pm}_{\Delta t}(\xi_n)$ scales linearly with Δt.

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is hard to take, since $f^{\pm}_{\Delta t}(\xi_n)$ scales linearly with Δt.

But we believe that the limit exists.

Workaround

- Re-write our discrete time update equation as:

$$\Delta x^{\pm}_{t_n}(\gamma) = w(t_n) + F(t_n, \gamma, x^{\mp}_{t_{n-1}}(\gamma - 1) - x^{\pm}_{t_{n-1}}(\gamma))$$

Workaround

- Re-write our discrete time update equation as:

$$\Delta x^{\pm}_{t_n}(\gamma) = w(t_n) + F(t_n, \gamma, x^{\mp}_{t_{n-1}}(\gamma - 1) - x^{\pm}_{t_{n-1}}(\gamma))$$

- Define an auxiliary process, with simplified update equation:

$$\Delta y^{\pm}_{t_n}(\gamma) = F(t_n, \gamma, y^{\mp}_{t_{n-1}}(\gamma - 1) - y^{\pm}_{t_{n-1}}(\gamma))$$

Evidence accumulation in the click task

We proved that x_t may be recovered from y_t at any time, using a normalization argument.

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Evidence accumulation for unknown h

$$\frac{dy^{\pm}(\gamma)}{dt} = \sum_{i \in I, j \in J} \left(C_{01}^{\pm} \delta_{t_{01}^{j}} + C_{10}^{\pm} \delta_{t_{10}^{i}} \right) + \frac{\gamma + \alpha - 1}{t + \beta} e^{y_t^{\mp}(\gamma-1) - y_t^{\pm}(\gamma)} - \frac{\gamma + \alpha}{t + \beta}$$

Research questions

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- Are our generative models plausible in biology?
- How do animals perform compared with our ideal-obs models?
- What are the rates of convergence of our posteriors?
- When do we encounter identifiability problems?
- What algorithm does the brain implement?

Bibliography I