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Evidence accumulation and change rate inference in dynamic environments Joint lab meeting

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> > 08/17/17

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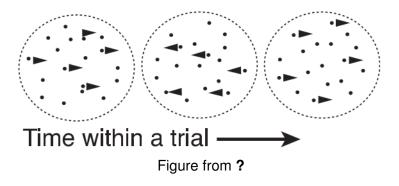
Modeling perceptual decisions



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Random dots reversal task



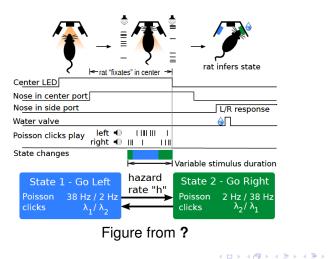
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Hidden Markov Model

HMM1.png

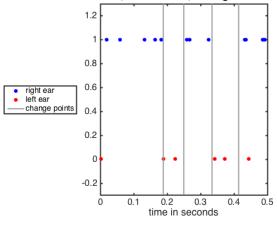
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Example trials



clicks; hazard= 5 Hz; low/high=2/38 Hz

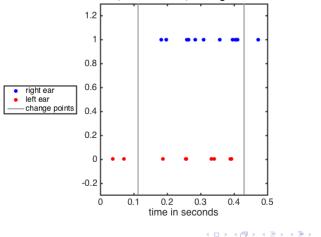
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lathematical theory Results Conclusion Modeling perceptual decisions Tasks

Example trials



clicks; hazard= 5 Hz; low/high=14/26 Hz

Adrian E. Radillo Inference in Perceptual Decisions

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Hidden Markov Models Evidence accumulation

Evidence accumulation

• decision variable, $y_t = \ln P(H^+|\xi_{1:t})/P(H^-|\xi_{1:t})$, represents accrued evidence in favor of either choice.

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Hidden Markov Models Evidence accumulation

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Hidden Markov Models Evidence accumulation

Evidence accumulation

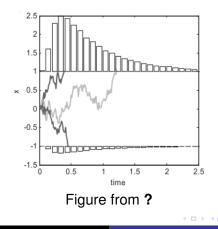
- decision variable, $y_t = \ln P(H^+|\xi_{1:t})/P(H^-|\xi_{1:t})$, represents accrued evidence in favor of either choice.
- decision rule:
 - free response: decide when threshold is reached, $|y_t| \ge \theta$
 - *interrogation*: decide at time T based on the sign of y_t

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Hidden Markov Models Evidence accumulation

Evidence accumulation

Static environment: $H_t \equiv \text{const.} \Rightarrow \text{weight all observations}$ equally



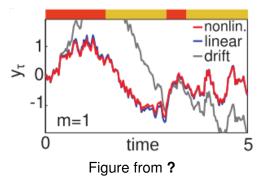
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Hidden Markov Models Evidence accumulation

Evidence accumulation

Dynamic environment: $\{H_t\}$ is a Markov Chain \Rightarrow discount old evidence



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Hidden Markov Models Evidence accumulation

How to deal with unknown hazard rate?

 \rightarrow track change point count a_t

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Hidden Markov Models Evidence accumulation

How to deal with unknown hazard rate?

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Recursive update equation -?

$$P_n\left(H^{\pm},a\right) \propto P\left(\xi_n | H^{\pm}\right) \left[\left(1 - \hat{h}_{n-1}(a)\right) \cdot P_{n-1}\left(H^{\pm},a\right) \\ + \hat{h}_{n-1}(a-1) \cdot P_{n-1}\left(H^{\mp},a-1\right) \right]$$

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Hidden Markov Models Evidence accumulation

Continuous-time approximation

Set:
$$\bar{P}_t^{\pm} := \sum_a P_n(H^{\pm}, a)$$
 and $\bar{A}_t^{\pm} := \frac{1}{t+\beta} \sum_a (a+\alpha) P_n(H^{\pm}, a)$

Moment closure - ?

$$\begin{split} d\bar{P}_t^{\pm} &= \bar{P}_t^{\pm} \left[\left(g^{\pm}(t) + \frac{1}{2} \right) dt + dW^{\pm} \right] + \left[\bar{A}_t^{\mp} - \bar{A}_t^{\pm} \right] dt \\ d\bar{A}_t^{\pm} &= \bar{A}_t^{\pm} \left[\left(g^{\pm}(t) + \frac{1}{2} \right) dt + dW^{\pm} \right] \\ &+ \left(\bar{A}_t^{\mp} - \bar{A}_t^{\pm} \right) \left(\frac{1}{t+\beta} + \bar{A}_t^{\mp} + \bar{A}_t^{\pm} \right) dt \end{split}$$

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Hidden Markov Models Evidence accumulation

Click task

Evidence accumulation ODE - known *h*

$$\frac{dy_t}{dt} = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2h \sinh(y_t)$$

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Hidden Markov Models Evidence accumulation

Click task

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Hidden Markov Models Evidence accumulation

But with unknown hazard rate?

• The state variable is a *2-state* continuous-time Markov Chain: $\{H_t\}_{t\geq 0}$



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Hidden Markov Models Evidence accumulation

But with unknown hazard rate?

- The state variable is a *2-state* continuous-time Markov Chain: $\{H_t\}_{t\geq 0}$
- One discretization approach is to partition the time line into N windows of width Δt

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Hidden Markov Models Evidence accumulation

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- One discretization approach is to partition the time line into N windows of width Δt
- In each Δt -bin, an observation is $\xi_t \in \{00, 01, 10, 11\}$

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Hidden Markov Models Evidence accumulation

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- The state variable is a *2-state* continuous-time Markov Chain: $\{H_t\}_{t\geq 0}$
- One discretization approach is to partition the time line into N windows of width Δt
- In each Δt -bin, an observation is $\xi_t \in \{00, 01, 10, 11\}$

Likelihoods summary

$$\begin{split} f^+_{\Delta t}(01) &= f^-_{\Delta t}(10) = \lambda_{\mathsf{high}} \Delta t + o(\Delta t) \\ f^+_{\Delta t}(10) &= f^-_{\Delta t}(01) = \lambda_{\mathsf{low}} \Delta t + o(\Delta t) \\ f^+_{\Delta t}(11) &= f^-_{\Delta t}(11) = o(\Delta t) \\ f^+_{\Delta t}(00) &= f^-_{\Delta t}(00) = 1 - (\lambda_{\mathsf{low}} + \lambda_{\mathsf{high}}) \Delta t + o(\Delta t) \end{split}$$

Hidden Markov Models Evidence accumulation

Revisit discrete-time update equation

Set
$$x_{t_n}^{\pm}(a) := \log P_n(H^{\pm}, a)$$
, and,

$$\hat{h}_n(a) := \frac{\alpha + a}{\beta + \Delta t \cdot n} = \frac{\alpha + a}{\beta + t_n}$$

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Hidden Markov Models Evidence accumulation

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, and,
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Update equation

$$\Delta x_{t_n}^{\pm}(a) = \log \frac{P(\xi_{1:n-1})}{P(\xi_{1:n})} + \log f_{\Delta t}^{\pm}(\xi_n) \cdots + \Delta t \cdot \hat{h}_{n-1}(a-1) e^{x_{t_{n-1}}^{\mp}(a-1) - x_{t_{n-1}}^{\pm}(a)} - \Delta t \cdot \hat{h}_{n-1}(a)$$

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Hidden Markov Models Evidence accumulation

Problematic continuum limit

When ξ_n corresponds to a single click (10,01), the limit,

$$\lim_{\Delta t \to 0} \left(\log \frac{P(\xi_{1:n-1})}{P(\xi_{1:n})} + \log f_{\Delta t}^{\pm}(\xi_n) \right),$$

is hard to take, since $f_{\Delta t}^{\pm}(\xi_n)$ scales linearly with Δt .

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Hidden Markov Models Evidence accumulation

Problematic continuum limit

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is hard to take, since $f_{\Delta t}^{\pm}(\xi_n)$ scales linearly with Δt .

But we believe that the limit exists.

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Hidden Markov Models Evidence accumulation

Workaround

• Re-write our discrete time update equation as:

$$\Delta x_{t_n}^{\pm}(\gamma) = w(t_n) + F(t_n, \gamma, x_{t_{n-1}}^{\mp}(\gamma - 1) - x_{t_{n-1}}^{\pm}(\gamma))$$

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Hidden Markov Models Evidence accumulation

Workaround

Re-write our discrete time update equation as:

$$\Delta x_{t_n}^{\pm}(\gamma) = w(t_n) + F(t_n, \gamma, x_{t_{n-1}}^{\mp}(\gamma - 1) - x_{t_{n-1}}^{\pm}(\gamma))$$

• Define an auxiliary process, with simplified update equation:

$$\Delta y_{t_n}^{\pm}(\gamma) = F(t_n, \gamma, y_{t_{n-1}}^{\mp}(\gamma - 1) - y_{t_{n-1}}^{\pm}(\gamma))$$

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Hidden Markov Models Evidence accumulation

Evidence accumulation in the click task

We proved that x_t may be recovered from y_t at any time, using a normalization argument.

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Hidden Markov Models Evidence accumulation

Evidence accumulation in the click task

We proved that x_t may be recovered from y_t at any time, using a normalization argument.

Evidence accumulation for unknown h

$$\frac{dy^{\pm}(\gamma)}{dt} = \sum_{i \in I, j \in J} \left(C_{01}^{\pm} \delta_{t_{01}^j} + C_{10}^{\pm} \delta_{t_{10}^i} \right) + \frac{\gamma + \alpha - 1}{t + \beta} e^{y_t^{\mp}(\gamma - 1) - y_t^{\pm}(\gamma)} - \frac{\gamma + \alpha}{t + \beta}$$

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Click task

Research questions

• Are our generative models plausible in biology?

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Click task

Research questions

- Are our generative models plausible in biology?
- How do animals perform compared with our ideal-obs models?

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Click task

Research questions

- Are our generative models plausible in biology?
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- What are the rates of convergence of our posteriors?

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Click task

Research questions

- Are our generative models plausible in biology?
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Click task

Research questions

- Are our generative models plausible in biology?
- How do animals perform compared with our ideal-obs models?
- What are the rates of convergence of our posteriors?
- When do we encounter identifiability problems?
- What algorithm does the brain implement?

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Click task

Bibliography I

Adrian E. Radillo Inference in Perceptual Decisions

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