UNIVERSITY of HOUSTON

Integrating evidence in a changing environment

INT workshop (Marseille)

Adrian E. Radillo
adrian@math.uh.edu
https://math.uh.edu/~adrian/presentations.html

06 April 2018

Acknowledgements


Alan Veliz-Cuba


Zachary P. Kilpatrick


Kresimir Josic

And also

Joshua I. Gold

Alex Piet

Perceptual decision making in real life

- Youtube video: dragonfly prey catching
  `https://youtu.be/XWROwMxepoM`

Some definitions for us

Perceptual decision-making

An animal engages in a behavior while having the possibility to choose *other* behaviors, and this decision is based on sensory evidence.

Some definitions for us

Perceptual decision-making

An animal engages in a behavior while having the possibility to choose *other* behaviors, and this decision is based on sensory evidence.

Ideal-observer model

Mathematical model of perceptual decision-making that meets some criteria of optimality.

Some definitions for us

Perceptual decision-making

An animal engages in a behavior while having the possibility to choose *other* behaviors, and this decision is based on sensory evidence.

Ideal-observer model

Mathematical model of perceptual decision-making that meets some criteria of optimality.

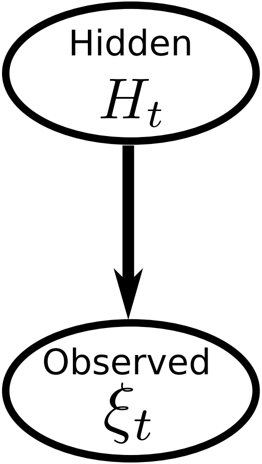
Optimality

Maximize reward

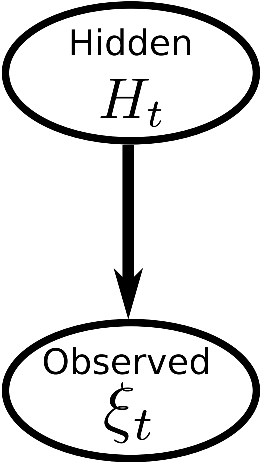
Disclaimer

We make a LOT of assumptions in the forward problem!

The filtering framework

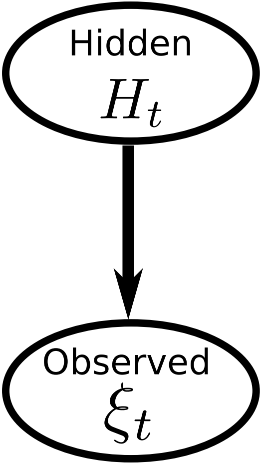


The filtering framework



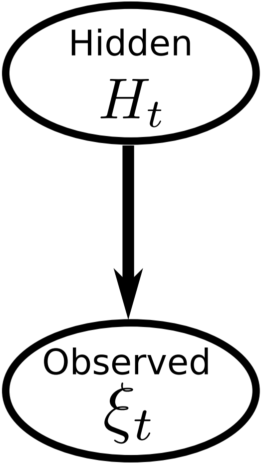
- Compute the posterior $P(H_T | \{\xi_t : 0 < t \leq T\})$

The filtering framework



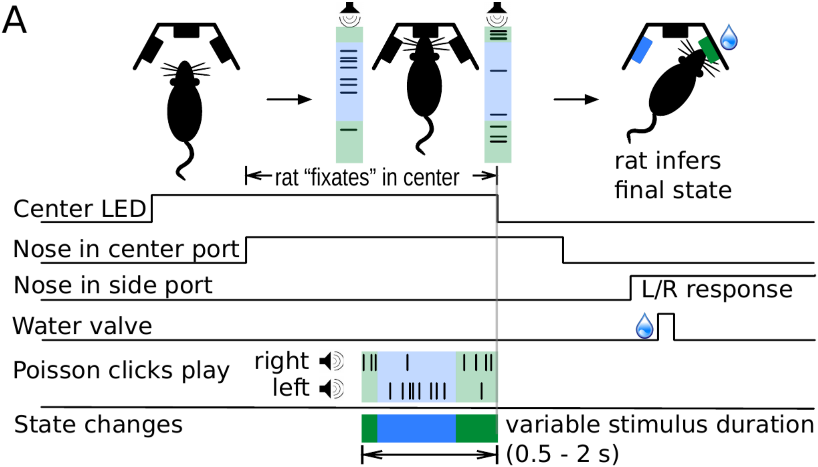
- Compute the posterior $P(H_T | \{\xi_t : 0 < t \leq T\})$
- If $\{H_t\}$ is Markov, we are in the Hidden Markov Model framework

The filtering framework



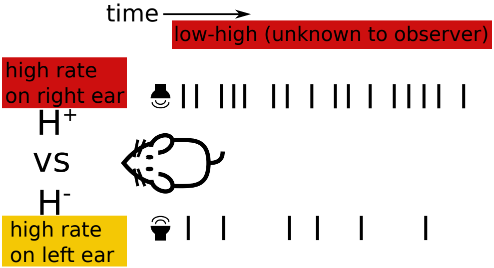
- Compute the posterior $P(H_T | \{\xi_t : 0 < t \leq T\})$
- If $\{H_t\}$ is Markov, we are in the Hidden Markov Model framework
- Applications of the filtering problem span a wide range of disciplines: Engineering, Finance, Genetics and many more!

Dynamic clicks task

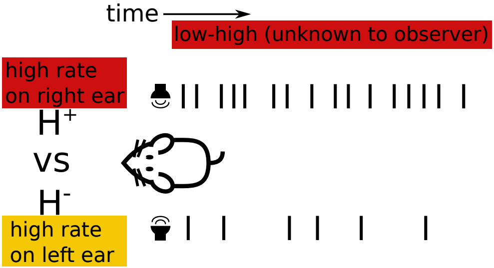


Piet, Hady, & Brody, (2017). Rats optimally accumulate and discount evidence in a dynamic environment. bioRxiv.

Static environment



Static environment



Log-likelihood ratio:

$$y_t := \log \frac{P(H(t) = H^+ | \xi_{[0,t]})}{P(H(t) = H^- | \xi_{[0,t]})}$$

Static environment

time →

low-high (unknown to observer)

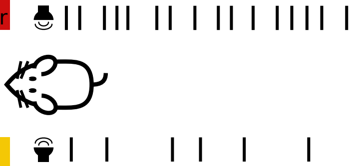
high rate on right ear

H+

vs

H-

high rate on left ear

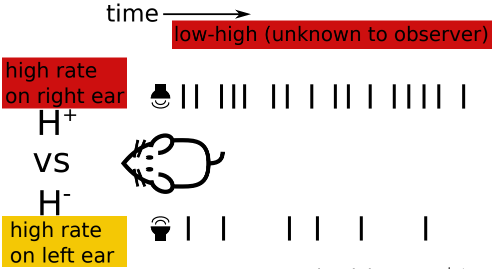


Log-likelihood ratio:

$$y_t := \log \frac{P(H(t) = H^+ | \xi_{[0,t]})}{P(H(t) = H^- | \xi_{[0,t]})}$$

Then $y_{t+\Delta t} = y_t + \log \frac{P(H(t) = H^+ | \xi_{[t,t+\Delta t]})}{P(H(t) = H^- | \xi_{[t,t+\Delta t]})}$

Static environment



Log-likelihood ratio:

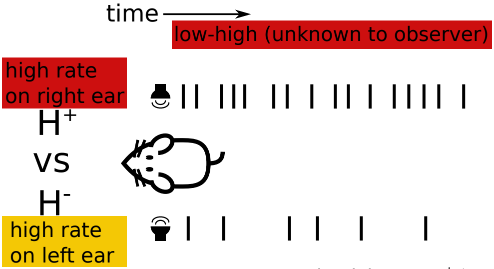
$$y_t := \log \frac{P(H(t) = H^+ | \xi_{[0,t]})}{P(H(t) = H^- | \xi_{[0,t]})}$$

Then $y_{t+\Delta t} = y_t + \log \dfrac{P(H(t) = H^+ | \xi_{[t,t+\Delta t]})}{P(H(t) = H^- | \xi_{[t,t+\Delta t]})}$

If no click in $[t, t+\Delta]$: $y_{t+\Delta t} = y_t$

If right (left) click in $[t, t+\Delta]$: $y_{t+\Delta t} = y_t \pm \log \dfrac{\lambda_{high}}{\lambda_{low}}$

Static environment



Log-likelihood ratio:

$$y_t := \log \frac{P(H(t) = H^+ | \xi_{[0,t]})}{P(H(t) = H^- | \xi_{[0,t]})}$$

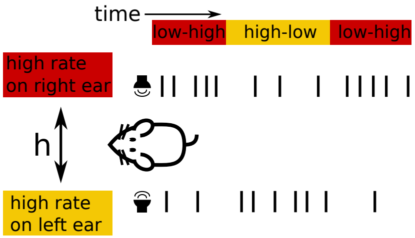
Then $y_{t+\Delta t} = y_t + \log \dfrac{P(H(t) = H^+ | \xi_{[t,t+\Delta t]})}{P(H(t) = H^- | \xi_{[t,t+\Delta t]})}$

If no click in $[t, t+\Delta]$: $y_{t+\Delta t} = y_t$

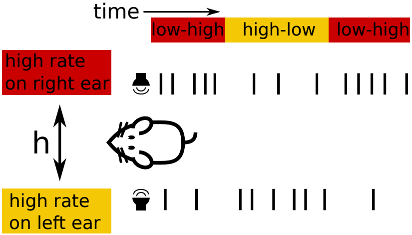
If right (left) click in $[t, t+\Delta]$: $y_{t+\Delta t} = y_t \pm \log \dfrac{\lambda_{high}}{\lambda_{low}}$

Then $dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^i) - \delta(t - t_L^j) \right)$, where $\kappa = \log \dfrac{\lambda_{high}}{\lambda_{low}}$

Changing environment: SDE

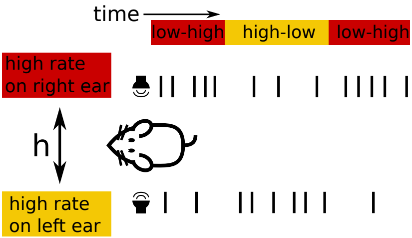


Changing environment: SDE

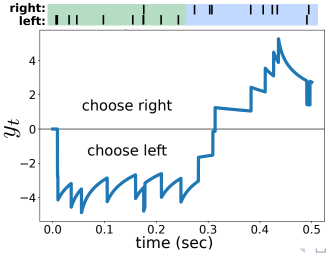


$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2h \sinh(y_t), \quad \kappa = \log \frac{\lambda_{high}}{\lambda_{low}}$$

Changing environment: SDE



$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2h \sinh(y_t), \quad \kappa = \log \frac{\lambda_{high}}{\lambda_{low}}$$



Changing environment: Signal-to-noise ratio

Signal at time t: Right clicks count - Left clicks count at time t

Changing environment: Signal-to-noise ratio

Signal at time t: Right clicks count - Left clicks count at time t

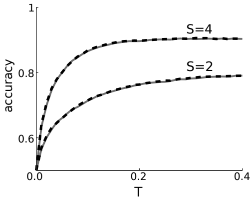
$$SNR(t) = \frac{E(N_R(t) - N_L(t))}{Stdev(N_R(t) - N_L(t))} = \sqrt{t}\frac{\lambda_{high} - \lambda_{low}}{\sqrt{\lambda_{high} + \lambda_{low}}} =: \sqrt{t} \cdot S$$

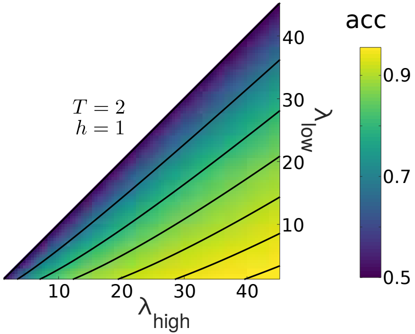
Changing environment: Signal-to-noise ratio

Signal at time t: Right clicks count - Left clicks count at time t

$$SNR(t) = \frac{E(N_R(t) - N_L(t))}{Stdev(N_R(t) - N_L(t))} = \sqrt{t}\frac{\lambda_{high} - \lambda_{low}}{\sqrt{\lambda_{high} + \lambda_{low}}} =: \sqrt{t} \cdot S$$

If $SNR(T)$ and $\sqrt{h}/S$ are kept constant, then accuracy at time T is constant.





Changing environment: Is nonlinearity needed?

$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2\sinh(y_t)$$

vs

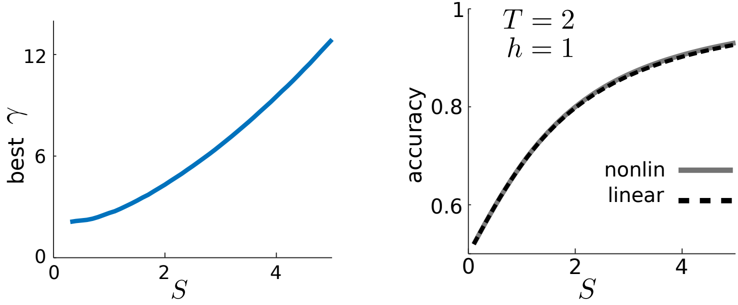
$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - \gamma y_t, \quad \text{for some } \gamma$$

Changing environment: Is nonlinearity needed?

$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - 2\sinh(y_t)$$

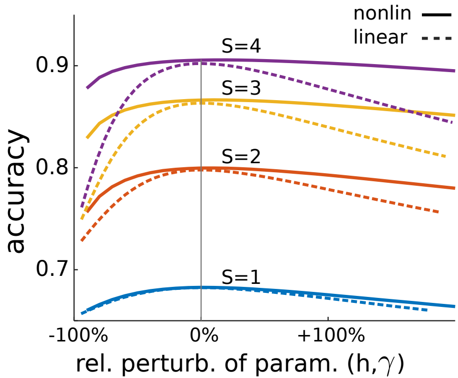
vs

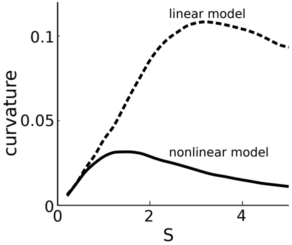
$$dy_t = \kappa \sum_{i \in I, j \in J} \left(\delta(t - t_R^j) - \delta(t - t_L^i) \right) - \gamma y_t, \quad \text{for some } \gamma$$



Linear model closely matches nonlinear model

Linear model is not as robust





Conclusion

- In a changing environment (with our assumptions) the optimal leak rate is nonlinear

Conclusion

- In a changing environment (with our assumptions) the optimal leak rate is nonlinear
- A linear leak reaches equivalent accuracy but is less robust to parameter tuning

Conclusion

- In a changing environment (with our assumptions) the optimal leak rate is nonlinear
- A linear leak reaches equivalent accuracy but is less robust to parameter tuning
- Accuracy of our model in the dynamic clicks task seems to only be governed by 2 parameters (instead of 4!)

Future work

- Learning h optimally is mathematically intractable $\rightarrow$ We need approximate algorithms

Future work

- Learning *h* optimally is mathematically intractable $\rightarrow$ We need approximate algorithms
- We need to tackle the *inverse problem*: Given data, how do we figure out what model was used?